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# British Mathematical Olympiad

**Round 1 : Friday, 1 December 2006**

**Time allowed** $3\frac{1}{2}$ hours.

**Instructions**

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

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# 2006/7 British Mathematical Olympiad
## Round 1

1. Find four prime numbers less than 100 which are factors of $3^{32} - 2^{32}$.

2. In the convex quadrilateral $ABCD$, points $M, N$ lie on the side $AB$ such that $AM = MN = NB$, and points $P, Q$ lie on the side $CD$ such that $CP = PQ = QD$. Prove that
$$\text{Area of } AMCP = \text{Area of } MNPQ = \frac{1}{3} \text{ Area of } ABCD.$$

3. The number 916238457 is an example of a nine-digit number which contains each of the digits 1 to 9 exactly once. It also has the property that the digits 1 to 5 occur in their natural order, while the digits 1 to 6 do not. How many such numbers are there?

4. Two touching circles $S$ and $T$ share a common tangent which meets $S$ at $A$ and $T$ at $B$. Let $AP$ be a diameter of $S$ and let the tangent from $P$ to $T$ touch it at $Q$. Show that $AP = PQ$.

5. For positive real numbers $a, b, c$, prove that
$$(a^2 + b^2)^2 \geq (a + b + c)(a + b - c)(b + c - a)(c + a - b).$$

6. Let $n$ be an integer. Show that, if $2 + 2\sqrt{1 + 12n^2}$ is an integer, then it is a perfect square.